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Magnetic fluid mechanics: a report on an International Advanced Course and Workshop

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An International Advanced Course on the Thermomechanics of Magnetic Fluids was held from 3 to 7 October 1977 at the International Centre for Mechanical Sciences (CISM), Udine, Italy, under the direction of B. Berkovsky. The course was a joint CISM/UNESCO undertaking and its general objectives were to assess knowledge in the field, to identify specific problem areas and promising directions for future research and applications, and to assist in the establishment of active relations between national research groups. It was the first conference to bring together the diverse and dispersed community of researchers in the field; there were 34 participants from 16 countries and 19 papers were included in the programme.

1. Introduction

Until quite recently, the mechanics of continuous media dealt mainly with diamagnetic and paramagnetic fluids. The attention of researchers was not drawn to strongly magnetizable fluids with intrinsic magnetic moments, probably because such media did not exist, either naturally or as man-made products. The subject gained importance in the early 1960s when it became apparent that magnetic fluids could be prepared synthetically. Since then, technological interest in magnetic fluids has grown rapidly, thus encouraging the development of their theory, which has now evolved into a recognizable branch of the mechanics of continuous media. Reviews on the mechanics and thermodynamics of magnetic fluids are given by Rosensweig (1966*a*, 1971), Bashtovoi & Berkovsky (1973), Shliomis (1974), Khallafalla (1975) and others. Theoretical models of different magnetic media have been developed and successfully applied to the solution of many problems; for example, see Kaiser & Miskolczy (1970), Tarapov (1972), Suyazov (1972), Luikov & Berkovsky (1974), Moskowitz & Ezekiel (1975), Woodson & Melcher (1969) and Perry & Jones (1976).

The most typical representatives of magnetic fluids are ferromagnetic fluids. Ferrofluids are composed of ultramicroscopic ferromagnetic particles colloidally suspended in a carrier liquid (Bibik & Lavrov 1965; Rosensweig, Nestor & Timmins 1965). They have almost the same magnetic characteristics as a solid, but in many respects behave as liquid continua. Magnetic liquids can be confined, positioned, shaped and controlled by magnetic forces. Today, the diverse applications for ferrofluids include vacuum and high pressure rotary seals and feedthroughs, controlled lubricants and damping systems. Other applications whose feasibility is being studied include material separators, fluid printers, visualization and display systems, bearings, activators, sensors, switches, and magnetocaloric heat pipes and engines which rely on the variable magnetization of the fluids with temperature.

This report presents a summary of information and issues discussed at the conference together with some clarifying background information.

2. New data on the magnetic fluids

The magnetic fluids are synthetic and research directed at the preparation and characterization of these fluids is an active topic. The small magnetic particles that give a magnetic fluid its magnetic property should preferably remain separate. A discussion of the forces between the particles (P. C. Sholten, Philips, Eindhoven) indicated under what conditions this can be accomplished. Monomolecular coatings on the surface of the magnetic particles serve as bumpers and overcome the tendency of the particles to clump together under an intense magnetic field. Rules for the choice of good coating molecules specify solvation of the molecule tails as indicated by exothermic mixing, and the presence of a polar head that adsorbs tenaciously onto the particle surface. Sidegroups or kinks in the tail prevent crystallization of the coating molecules and are therefore favourable. Comparison of the attractive van der Waals and magnetic forces between particles with the steric repulsion force due to the presence of the coating molecules leads to a picture of particle stability and, when it occurs, agglomeration: clusters are sometimes formed in the absence of an applied field owing to the permanent magnetization of the single-domain particles, but the clusters possess magnetic closure with little external field; when subjected to an applied magnetic field, the clusters polarize and may agglomerate with neighbours.

Interaction between monodomain ferromagnetic grains suspended in the carrier fluid was also the subject of a lecture by P. Pincus (University of California, Los Angeles). Thus dipolar induced aggregations of ferromagnetic grains may be analysed in the low density regime, where pair correlations dominate. The pair correlations are generally related to the second virial coefficient, the integrated ratio of dipole–dipole interaction energy to thermal energy. In the limit of strong coupling with intense external fields the dipolar energy is minimized in a head-to-tail configuration leading to chain formation. An expression may be found for the number of grains per chain and the mean-square extent perpendicular to the field.

In the absence of an applied field the situation is more delicate and the grain shape plays an important role. For ideally spherical particles with no external field the chain formation still seems to occur (de Gennes & Pincus 1970; Jordan 1973) in the form of partially flexible or wormlike chains. In a weak field the chains are predicted to elongate along the field provided that Jacobs–Bean ring formation is negligible. With long needles the antiparallel arrangement is more stable and may account for observations of spherical clusters in certain systems. Ultimately one desires a complete theoretical understanding of the ferrofluid diagram as a function of particle concentration, temperature and magnetic field.

In considering dynamic influences on magnetization of the grains it is necessary to distinguish superparamagnetic behaviour in which magnetization is relatively decoupled from the crystal axes and cases in which the crystalline anisotropy energy is large. In the former case, the particle is stationary while the magnetization field vector rotates when the fluid containing the particles is subjected to a rotating external field. In the latter case, motion of the particle and the magnetization vector are locked together and the rotational motion produces viscous dissipation within the fluid. Observation of non-mechanical torque-driven flow of a ferromagnetic fluid was first reported by Moskowitz & Rosensweig (1967), with swirl flow induced in the magnetic fluid by imposing a relatively uniform rotating external field. Introducing separate equations of motion for magnetization and the orientation of an isolated grain determines the phase angles between the field direction, the easy crystal axis and the particle angular velocity. When the rotating field cannot supply sufficient power to balance the viscous dissipation the particle crystallographic axis tips out of the plane and precesses. When considering the flow surrounding a grain of the ferrofluid a number of effects and processes such as Brownian motion which may be ignored at larger length scales become important. The study of flow systems with such small length scales is a relatively new field currently being developed (Batchelor 1976).

Pulse magnetization provides a means of detecting whether magnetization is locked to the grain crystal axis or is able to orient freely. Measurements with a pulsed-field inductive magnetometer that measures the magnetic moment of a ferrofluid as a function of time were reported by E. H. Bogardus (IBM, Yorktown Heights). After the applied field has been removed from various samples of water-based ferrofluids the magnetization is characterized by a fast decay over less than 1 μs, superimposed with a gradual decay that can be as long as 4 ms. The fast component of magnetization varies from about half to nearly the total magnetization and is due to magnetization processes within the grain. The slow component is believed to be due to particle rotation and can be related to particle size. This study of the dynamic magnetization of the ferrofluids resulted from the development of rapid, non-impact printing using magnetizable ferrofluid ink.

E. A. Peterson (USAF Academy, Colorado) presented experimental studies concerning field-induced agglomeration in magnetic colloids. The clustering is pronounced for present-day water-based ferrofluids and nearly absent for many other compositions such as a well-stabilized ferrofluid in a diester or hydrocarbon carrier fluid. The ferrofluids are placed in a vertical tube and subjected to an applied magnetic field inducing particle clustering, the clusters redistributing under gravity in the tube by sedimentation and the concentration being detected with an inductance sensor. The clustering is reversed by removal of the field, thermal agitation acting to redisperse the agglomerates. This technique should be useful in evaluating the stability of new ferrofluid compositions.

In the absence of an applied magnetic field a magnetic colloid behaves like an isotropic fluid. The application of a magnetic field establishes a preferred direction breaking the symmetry. The resulting anisotropy should affect macroscopic properties and this topic was the subject of a lecture by A. Martinet (Université Paris XI, Orsay), who discussed anisotropy in both static and rotating fields.

In studying the anisotropy phenomena Martinet developed methods of polymerizing the fluid matrix, thereby trapping the grains. The organic carrier toluene or kerosene is replaced with a polymer of styrene and divinylbenzene. For water-based ferrofluids the matrix substitution is accomplished with a polyvinyl alcohol. This artifice makes it possible to study anisotropy with and without structural changes due to clustering.

Thus de Gennes & Pincus (1970) introduced a critical parameter $\lambda = \mu^2/a^3kT$, the coupling coefficient (where μ is the dipole moment of a particle and a is the minimum approach distance), which compares the magnetic interaction eneigy μ^2/a^3 with the thermal energy kT. Chaining of the grains is expected to appear for $\lambda > 1$. From Martinet's photomicrographs high chaining takes place for a toluene-based cobalt fluid ($\lambda \approx 2$) as well as for a water-based magnetite fluid ($\lambda \approx 1$) while no perceptible chaining is directly observable for a kerosene-based magnetite fluid in which $\lambda \approx 0{\cdot}2$. This behaviour is evidenced also in measurements of magnetization, wherein the initial susceptibility is increased several fold in fluid samples relative to polymerized samples (except for the kerosene ferrofluid) and so conclusively demonstrates the influence of structural changes. Dynamic anisotropy was demonstrated by measurement of the perpendicular and parallel components of magnetization of ferrofluids under rotation, before and after polymerization.

Given these properties of magnetic fluids, the analyst can begin to construct more complete hydrodynamic models that take into account the two-phase nature of the ferromagnetic medium.

The technique of X-ray small angle scattering was reported to be suitable for the *in situ* study of particles in the ferrofluids (C. Petipas, University of Rouen, Mont St Aignan), the scattering being related to the existence of heterogeneities in the electronic density of the matter. These studies determined the size and size distribution of the particles, the stability of the fluid in time, and the grain correlations in the absence and in the presence of a magnetic field. Another exploratory study by Petipas using neutron small angle scattering suggests that the thickness of the surfactant coating on the surface of the grains may also be deduced; the technique depends on scattering by the atomic nucleus with a complication due to scattering of a magnetic origin. The influence of magnetic-fluid anisotropy on optical properties was also discussed by R. V. Mehta (Sardar Vallabhbhai College, Surat).

All the existing ferrofluids use organic solvents on water as the liquid carrier but liquid metals are attractive candidates for this role. The high thermal conductivity would permit very high speed rotary seals in which viscous-generated heat is readily conducted out of the fluid; high conductivity could also help to miniaturize magnetocaloric energy converters, in which heat energy added to a ferrofluid in the presence of an applied magnetic field is converted directly into mechanical power of the flowing stream. S. W. Charles (University College of North Wales, Bangor), who is interested in the energy conversion problem, reported on the preparation and characterization of metallic ferrofluids composed of iron particles suspended in mercury. The addition of sodium prevents high temperature diffusional growth of relatively large particles at the expense of smaller particles, which possess a greater solubility. At present, these suspensions settle rapidly under the attraction of a magnetic field gradient, expelling a portion of the mercury carrier that is particle free. What is missing in the make-up of these suspensions is a repulsive mechanism to prevent the strong agglomeration of particles with each other; exploitation of work-function differences may permit electric-charge stabilization in the future.

The present prospects of synthesizing advanced ferrofluids rely almost entirely on further optimization of the colloidal solutions of magnetic grains in a carrier fluid and there is a great opportunity for improvements. However Andres (1976) recently reported practical levitation results using aqueous molecular solutions of paramagnetic salts which gave a magnetization $B-H$ greater than 29 gauss in an applied field of 10^4 oersteds. In another direction, the statistical-mechanical study of Hemmer & Imbro (1977) is tantalizing in defining the values of the molecular parameters of a liquid required in order for ferromagnetization to appear. On a theoretical basis liquid ferromagnetization occurs if the exchange interaction is sufficiently strong.

3. Towards applications of magnetic fluids

It is clear that ferrofluids represent a new class of magnetizable liquids with interesting properties capable of having a substantial impact on technology. In tune with this observation, applications of ferromagnetic liquids were surveyed in the paper by M. P. Perry (G. E. Corporate Research Centre, Schenectady), which emphasized the best developed ideas, as well as those applications which show signs of displacing more conventional techniques. The areas of application noted were ferrofluid sealing, hydrodynamic bearings and lubrication, medical applications, energy conversion, fluidmagnetic buoyancy, and others including magnetic-ink non-impact printers and magnetic-fluid damping.

Concerning surgical applications, Roth (1969) reported experiments where an aneurysm was successfully blocked off from a blood vessel using a static ferrofluid plug. More recently it has been shown that veins and arteries in experimental animals can be successfully plugged *in vivo* using a ferrofluid. However, the principal sealing application of ferrofluids at present is in rotary-shaft seals, which were developed and are marketed by a private company, Ferrofluidics Corporation (Burlington, Massachusetts), and have been studied by various industrial and university laboratories. The theory and application of these ferrofluid seals were the subject of a lecture by T. B. Jones (Colorado State University, Fort Collins). The theory of the seals may be conveniently divided into three topics: the formulation of forces in ferrofluids, solution of various ferrohydrostatic equilibria and consideration of static ferrofluid seals and plugs.

Magnetic-fluid seals operated under design conditions last indefinitely with no mechanical wear, since the parts do not touch each other. As such, the seals constitute a unique and useful machine element. Practical magnetic-fluid rotary-shaft seals use discrete rings of the liquid magnetically held in position between a rotary shaft and a series of annular teeth surrounding the shaft. Magnetic flux is focused by the teeth, which are made of magnetically permeable alloy, the concentrated flux crossing the gap into the permeable shaft. The high localized gradients of magnetic field produce forces that position the liquid in any given ring. In addition, the ring of liquid supports a difference in pressure across it through a slight displacement and deformation. R. L. Bailey (University of Oxford) has studied rings at the bursting point of a seal. Bursting occurs when a pinhole of sealed gas reaches the far liquid surface, and may be observed advancing through the liquid with increasing pressure. Provided that the pressure of the sealed gas is relieved by leakage through the pinhole breach or higher pressure is built up on the far side, rehealing occurs. This mechanism is important in the staging of the seal to accommodate a greater total pressure than a single stage can withstand. It is found that a drop at about 25 % below the bursting pressure is sufficient to allow resealing of a given stage of the seal.

A good seal fluid has little tendency to cluster, i.e. the coupling coefficient λ is small. A seal assembly taken from storage will display an increased starting torque due to particle migration in the fluid and the increased viscosity that accompanies increased particle loading. But a rotation of the seal shaft redisperses the concentration variations and results in a low running torque with no leakage of sealed gas at any time during the start-up or running process. This occurs despite the fact that the local body force on the fluid in a seal is of the order of 500 000 times the force of gravity and testifies to the near-ideal performance that can be achieved with a good design even if the fluid is non-ideal.

The fluid-mechanical properties of magnetic fluids also lead to novel fluid bearings of several generic types (R. E. Rosensweig, Exxon Corporate Research Laboratories, Linden). Passive bearings, which use no source of applied energy, escape the limitations of Earnshaw's theorem and produce hydrostatic levitation of movable members while dynamic bearings, which use magnetic fluids, offer contactless support with no need to circulate the fluid, and hence no external pump.

Thus as early as 1839 Earnshaw propounded the theorem that stable levitation of isolated collections of charges (or poles) is not possible by static fields. Nonetheless, inventors continue to seek all repulsion combinations of magnets in order to float objects free of contact with any solid support. Such efforts are in fact doomed to failure. However, it is interesting to note that in 1938 Braunbeck deduced that diamagnetic materials and superconductors escape the restrictions underlying Earnshaw's theorem. These special materials may be successfully suspended although the diamagnetics suffer from a very low load support and superconductors require cryogenic refrigeration. More recently, the discovery was made first of the levitation of non-magnetic objects immersed in magnetic fluids subjected to an applied magnetic field and then of the self-levitation of immersed permanent magnets and the like with no external field present (Rosensweig 1966*a*, *b*, 1971). Subsequently, Rabenhorst and others disclosed the concepts of bi-fluid passive bearings in which a magnetic fluid is used as a seal to capture a pad of pressurized air or some other non-magnetic fluid medium which serves to support the load. In each of these instances Earnshaw's theorem is circumvented by incorporating an additional element into the physical situation.

The mechanism of single-fluid bearings is directly related to the influence of the magnetic field on the distribution of hydrodynamic pressure in the magnetic fluid. In the magnetic-field Bernoulli relationship the fluid–magnetic term $\mu_0\bar{M}\bar{H}$ is subtracted from the conventional sum of terms expressing pressure, gravitational and kinetic influences on the flow along a streamline. Detailed solutions reveal that the support stiffness is not always increased by an increase in the magnetic-fluid permeability as there are concomitant effects of magnetic flux bypassing and fluid–magnetic force production that compete with each other.

The study and use of magnetic-fluid bearings are in their infancy. To date, mostly prototype designs have seen the light of day: examples include inertial sensors of velocity or acceleration where sticking friction must be negligible, a spindle bearing adapted for textile fibre production that runs silently without wear and potentially for very long times, and a bearing intended to support high speed flywheels for energy storage. Commercial production of inertial dampers containing a floated mass for computer peripherals has begun, and the bearing mechanism comes into play in the commercial application of magnetic fluids in loudspeakers, wherein the fluid centres the voice coil magnetically to prevent mechanical rubbing and malfunction while simultaneously removing electrical heat.

4. Theoretical models and governing equations

Among the theoretical investigations reported at the Course, the central place was given to the analysis of magnetic-fluid models and the formulation of governing equations. This was certainly not without reason. A magnetic fluid is a complex, multi-component and multi-phase medium with essential fluidity and magnetic properties. A quantitative description of its thermomechanics is very difficult and requires a well-defined and justified theoretical model. Such a model should include the most important features but should at the same time be simple enough to handle. There has been a certain amount of progress in this direction. A fairly simple model adequately describing the steady or quasi-steady flow of magnetic fluids and convective heat transfer in slowly or slightly changing external magnetic fields has been developed. This was suggested in one of the first contributions by Neuringer & Rosensweig (1964) and has been widely used. The model considers a magnetic fluid as a homogeneous, isotropic and monophase medium with the magnetic moment parallel to the magnetic field and leads to governing equations which are slightly modified Navier–Stokes equations, with the addition of the Maxwellian stress tensor for a magnetic field in the fluid. There was a certain controversy in that alternative formulations of the magnetic body force acting on the magnetic fluid could be used within the model. However, it was recently found that different formulations of the magnetic force density, derived from different considerations, are practically equivalent. T. B. Jones presented a thorough review of the diverse body-force formulations. He stressed the fact that ambiguities in the force density arise in the case of incompressible media because of the arbitrariness in the definition of the hydrostatic pressure and this was illustrated for several particular problems for which a simple solution can be developed.

Further theoretical and experimental investigations have shown a certain narrowness in this model, mainly in that it leaves out possible rotation of suspended magnetic grains and the influence of such rotation on macroscopic motion. Attention has been directed to the fact that the external magnetic field affects magnetic particles not only through a force but also through a magnetic couple. The couple turns the magnetic particles and may set them rotating. For example, in a revolving magnetic field, suspended magnetic grains begin to spin and each grain becomes a centre of microscopic vorticity. The average effect of rotation of magnetic particles in certain conditions may cause a macroscopic flow. Some investigators believe that this is likely to happen when and where there is a well-pronounced non-uniformity in either the magnetic field or the properties of the liquid. The rotational degree of freedom of magnetic fluids can be accounted for consistently through the introduction of an internal spin field, essentially different from the angular velocity, defined as half a vorticity vector, and through the recognition of the existence of a pseudo-vector magnetic body couple and couple stress tensor. In this case, the stress tensor becomes asymmetrical and conservation of the total moment of momentum is not automatically fulfilled when that of the linear momentum is assured. Thus a system of governing equations of the thermomechanics of magnetic fluids includes an additional equation governing the variation of the internal angular momentum.

A critical analysis of the hypotheses on which the Neuringer–Rosensweig model of magnetic fluids is based and the development of an asymmetrical model were given by B. M. Berkovsky (UNESCO, Paris). The main emphasis was laid on the formulation of more correct governing equations taking into account the intrinsic angular momentum and the dynamics of magnetization as well as the feedback from microscopic motion of magnetic grains to the bulk of the magnetic fluid and its polarization. A specific difficulty in solving thermomechanical problems in the framework of the asymmetrical model of magnetic fluids was stressed; this difficulty stems from the fact that particular boundary conditions for the intrinsic angular velocities should be formulated. Two simple examples of boundary conditions were discussed: first, when rigid boundaries interact with the neighbouring magnetic fluid so strongly that it can no longer rotate freely and thus the intrinsic angular velocity of the magnetic fluid at the surface will be equal to that of the surface itself; second, when the neighbouring fluid is able to rotate freely in relation to the boundary surface and thus the density of the couple stress tensor will be equal to zero. Some hints on the formulation of boundary conditions between these limiting cases were given. Specific asymmetrical phenomena and also some more favourable circumstances for their observation were discussed. The attention of participants was drawn to the need for experimental and further theoretical investigations as regards unsteady and non-isothermal flows in essentially inhomogeneous magnetic fields.

J. Buckmaster (University of Illinois, Urbana) presented a particular example of the application of the Neuringer–Rosensweig model to a problem of boundary layers in ferromagnetic fluids. He deduced equations for the boundary-layer flow for a ferrofluid and solved them numerically for a variety of circumstances. It was shown that, under certain realistic assumptions, the equations are identical to those of an incompressible heat-conducting fluid except for the presence of a body-force term which depends on the gradient of the magnetic field strength and the variation of the magnetization across the layer. He pointed out that, under additional assumptions, these simplified equations are identical to those governing convection boundary layers, the magnetic force corresponding to the gravity force. This provides useful physical insight into the behaviour of ferroliquid boundary layers. Numerical calculations and other arguments show that the magnetic force can significantly delay or enhance separation. Indeed, Buckmaster showed that, if the force is unfavourable, the separation point can be moved all the way forwards to a front stagnation point, whereas, if the force is favourable, separation can be delayed to a point arbitrarily close to a rear stagnation point.

5. Instabilities and waves

For continuous media the investigation of instabilities and waves has always attracted great interest, and magnetic fluids have proved to be no exception. Active research started as soon as the existence of magnetic fluids became apparent. First, attention was focused on surface instabilities in isothermal fluids, the reason being their relatively simple theoretical exploration and experimentation (e.g. see Cowley & Rosensweig 1967; Zelazo & Melcher 1969). Attention was later focused on bulk instabilities, mainly convective instabilities in non-isothermal magnetic fluids (e.g. see Luikov & Berkovsky 1974). The main results obtained so far have considered magnetic fluids as homogeneous, incompressible, electrically non-conducting continuous media with constant transfer coefficients. Little work has been done on the stability problems for essentially non-uniform magnetic fields, complicated geometries and flows of magnetic fluids.

The main findings with regard to surface instabilities can be stated as follows: a perpendicular magnetic field has a destabilizing influence on a flat interface between a magnetizable and non-magnetizable fluid; a tangential field, without changing the surface stability threshold, may essentially influence the shape of the surface, yielding a new stable state. In order that instability will occur, the fluid's magnetization in response to the applied magnetic field should exceed certain critical values. As regards convective instability, it was found that a uniform transverse magnetic field has a stabilizing effect; a vertical magnetic field stabilizes an irregular perturbation only along its direction of motion, producing roll cells with axes parallel to the field while leaving the instability threshold unchanged. A non-uniform magnetic field qualitatively influences the stability of a plane parallel layer of magnetic fluid in the same way that a non-uniform temperature field brings about convective instability. A magnetic field with a gradient parallel to the gravity force decreases a heated layer's stability. If directed oppositely, the magnetic field gradient stabilizes the layer, and it has been found that for any temperature gradient a finite magnetic field gradient exists, which stabilizes the layer entirely.

A fairly detailed review of results resulting to convective and surface instabilities of non-isothermal horizontal layers of magnetic fluid was given in a paper by V. G. Bashtovoi (Luikov Institute for Heat and Mass Transfer, Minsk). As new aspects, combined surface–convective instability of magnetic fluid was described and allowance for magnetic-field perturbation was considered. It was shown that taking magnetic-field perturbation into account in general brings about an increase in stability. This phenomenon was illustrated for the well-known problem of the convective stability of a horizontal plane layer in a perpendicular magnetic field. It was underlined that, in the case of a non-isothermal magnetic-fluid layer, surface and convective instability are essentially related: instability of the free surface necessarily causes convection within a fluid. Stability criteria were derived and discussed within the framework of a linear theory. It was emphasized that combined surface–convective instability is a very interesting phenomenon which has not yet been sufficiently discussed.

R. E. Rosensweig, M. Zahn & T. Vogler (Exxon Research and Engineering Company, Linden, New Jersey) reported results on stabilization of fluid penetration through a porous medium using a magnetizable fluid. They concentrated on the investigation of the interface between two fluids one of which is more viscous and is driven through a cell by a less viscous magnetic fluid. A linear hydrodynamic analysis of the stability of a moving interface in a Hele–Shaw cell was made and this led to instability criteria. It was found that this interface can be stabilized for sufficiently small wavelengths by an imposed magnetic field. This result can also be applied to penetration through a porous medium. The authors described experiments undertaken and the quantitative comparison between theory and experiment. This presentation was accompanied by a colour film illustrating the development of finger instability and prevention of instability by the application of an external magnetic field.

A mathematical formulation of problems of instability and the development of infinitesimal waves on a plane interface between a ferrofluid and a non-magnetic fluid was communicated by J. P. Brancher (Laboratoire d'Energétique et Mécanique Théorique et Appliquée, Nancy). He also touched upon first-order nonlinear effects of surface wave propagation. His analysis was restricted to a simple case where a ferrofluid occupied the lower half-space and the initial magnetic field was uniform and vertical.

6. Conclusions

As hoped, the meeting made possible a comprehensive state-of-the-art assessment of research in the mechanics and physical chemistry of magnetic fluids and their engineering applications. It also greatly assisted in the establishment of active relations between different research groups. There can be no doubt that the forthcoming publication of the lectures and papers presented† will favour the exchange, diffusion and application of the most advanced knowledge in the field.

As regards recommended future research and development directions, it would seem that most of the participants felt that the main emphasis should be laid on the further improvement of the quality of magnetic fluids, the synthesis of new magnetic fluids and the development of technologies allowing the production of magnetic fluids with pre-defined properties. Methods of investigating the properties of magnetic fluids should also be further developed. It was felt that in the immediate future the attention of researchers should mostly be concentrated on experimental work, in order to provide the fastest progress. More experimental work is necessary on the investigation of flow structure, convection, instabilities, waves, etc. Among the theoretical investigations, priority should be given to those relating to applications and the development of more adequate mathematical models of magnetic fluids. New phenomena brought about by the body couples should also be systematically investigated.

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